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On Gauge-Invariant Decomposition of Nucleon Spin

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Abstract

We investigate the relation between the known decompositions of the nucleon spin into its constituents, thereby clarifying in what respect they are common and in what respect they are different essentially. The decomposition recently proposed by Chen et al. can be thought of as a nontrivial generalization of the gauge-variant Jaffe-Manohar decomposition so as to meet the gauge-invariance requirement of each term of the decomposition. We however point out that there is another gauge-invariant decomposition of the nucleon spin, which is closer to the Ji decomposition, while allowing the decomposition of the gluon total angular momentum into the spin and orbital parts. After clarifying the reason why the gauge-invariant decomposition of the nucleon spin is not unique, we discuss which decomposition is more preferable from an experimental viewpoint.

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I. INTRODUCTION

After years of theoretical and experimental efforts, it has been established that only about 1/3 of the nucleon spin comes from the intrinsic quark spin [1] -[4]. However, the following question still remains to be solved. What carries the remaining 2/3 of the nucleon spin ? Is it gluon polarization ? Or, is it orbital angular momentum of quarks and/or gluons ? When discussing the spin contents of the nucleon, however, one must be careful about the fact that the decomposition of the nucleon spin is not necessarily unique. (One should also keep in mind the scale-dependent nature of the nucleon spin contents [5] -[9].) There have been two popular decompositions of the nucleon spin. One is the Jaffe-Manohar decomposition [10], while the other is the Ji decomposition [11]. Recently, still another decomposition of the nucleon spin has been proposed and advocated by Chen et al. [12],[13]. A natural question is how are they mutually related and in what respect are they essentially different ?

Since the essential physics seems to be common in two gauge theories, i.e. QED and QCD, we start by explaining the problem in the former case, which is theoretically simpler. The most popular gauge-invariant angular-momentum decomposition of the coupled electron-photon system is given by

$$\begin{aligned} \boldsymbol{J}_{QED} &= \int \psi^\dagger \, \frac{1}{2} \, \boldsymbol{\Sigma} \, \psi \, d^3x \; + \; \int \psi^\dagger \, \boldsymbol{x} \times \frac{1}{i} \, \boldsymbol{D} \, \psi \, d^3x \; + \; \int \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B}) \, d^3x \\ &= \boldsymbol{S}^e \; + \; \boldsymbol{L}^e \; + \; \boldsymbol{J}^\gamma, \end{aligned} \tag{1}$$

with $\boldsymbol{D} \equiv \nabla - i \, e \, \boldsymbol{A}$ the covariant derivative. This corresponds to the Ji decomposition of the nucleon spin in the case of QCD [11]. An advantage of this decomposition is that each piece of the decomposition is separately gauge invariant. What is lacking, however, is a further decomposition of the total angular momentum of the photon into the intrinsic spin and orbital parts. The decomposition of $\boldsymbol{J}^\gamma$ into the spin and orbital parts is known to be made by adding to Eq.(1) a surface term

$$\int \nabla^j \, [\, E^j \, (\boldsymbol{x} \times \boldsymbol{A}\,) \,] \, d^3x, \tag{2}$$

which vanishes after integration. The result is well known :

$$\begin{aligned} \boldsymbol{J}_{QED} &= \int \psi^\dagger \, \frac{1}{2} \, \boldsymbol{\Sigma} \, \psi \, d^3x \; + \; \int \psi^\dagger \, \boldsymbol{x} \times \frac{1}{i} \, \nabla \, \psi \, d^3x \\ &+ \int \boldsymbol{E} \times \boldsymbol{A} \, d^3x \; + \; \int E^i \, \boldsymbol{x} \times \nabla \, A^i \, d^3x \\ &= \boldsymbol{S}^e \; + \; \boldsymbol{L}^{e\prime} \; + \; \boldsymbol{S}^{\gamma\prime} \; + \; \boldsymbol{L}^{\gamma\prime}. \end{aligned} \tag{3}$$

It corresponds to the Jaffe-Manohar decomposition of nucleon spin in the case of QCD [10]. It seems natural to identity the above four terms as the electron spin, electron orbital angular momentum, photon spin and photon orbital angular momentum, respectively. However, the problem is that, except for the electron spin part, all of the other terms in this decomposition are gauge dependent so that they have a obscure physical meaning.

A new proposal by Chen et al. appears to circumvent this difficulty [12],[13]. A principle idea is to decompose the vector potential $\boldsymbol{A}$ into the two parts, $\boldsymbol{A}_{pure}$ and $\boldsymbol{A}_{phys}$ satisfying the condition

$$\nabla \cdot \boldsymbol{A}_{phys} = 0, \quad \nabla \times \boldsymbol{A}_{pure} = 0. \tag{4}$$

As we shall see later in more detail, $\boldsymbol{A}_{phys}$ and $\boldsymbol{A}_{pure}$ are nothing but the transverse and longitudial components of the vector potential $\boldsymbol{A}$. By adding to Eq.(1) another surface term

$$\int \nabla^j \left[E^j \left(\boldsymbol{x} \times \boldsymbol{A}_{pure} \right) \right] d^3x, \tag{5}$$

they obtain a new decomposition of the angular momentum in QED :

$$\begin{aligned} \boldsymbol{J}_{QED} &= \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma} \psi \, d^3x \; + \; \int \psi^\dagger \, \boldsymbol{x} \times \frac{1}{2} \boldsymbol{D}_{pure} \psi \, d^3x \\ &+ \int \boldsymbol{E} \times \boldsymbol{A}_{phys} \, d^3x \; + \; \int E^j \left(\boldsymbol{x} \times \nabla \right) A^j_{phys} \, d^3x \\ &= \boldsymbol{S}^e \; + \; \boldsymbol{L}^{e\prime\prime} \; + \; \boldsymbol{S}^{\gamma\prime\prime} \; + \; \boldsymbol{L}^{\gamma\prime\prime}, \end{aligned} \tag{6}$$

where $\boldsymbol{D}_{pure} = \nabla - i \, e \, \boldsymbol{A}_{pure}$. A great advantage of this decomposition is that, while allowing the decomposition of $\boldsymbol{J}^\gamma$ into the spin and orbital parts, each of the four terms is separately gauge invariant. This statement can easily be checked by using the gauge transformation property of $\boldsymbol{A}_{pure}$ and $\boldsymbol{A}_{phys}$:

$$\boldsymbol{A}_{pure} \longrightarrow \boldsymbol{A}'_{pure} = \boldsymbol{A}_{pure} + \nabla \Lambda, \tag{7}$$

$$\boldsymbol{A}_{phys} \longrightarrow \boldsymbol{A}'_{phys} = \boldsymbol{A}_{phys}. \tag{8}$$

Another remarkable feature of this new decomposition is that, in a particular gauge, i.e. in the Coulomb gauge, in which $\boldsymbol{A}_{pure} = 0$ and $\boldsymbol{A} = \boldsymbol{A}_{phys}$, it is reduced to the decomposition (3), which corresponds to the Jaffe-Manohar decomposition in the QCD case.

Extending the analysis to the QCD case, Chen et al. derived a new gauge-invariant decomposition of the nucleon spin, which is a generalization of (6). As a byproduct of this

analysis, they come to propose and advocate a new gauge-invariant decomposition of the total linear momentum in QCD given as

$$\begin{aligned} \boldsymbol{P}_{QCD} &= \int \psi^\dagger \frac{1}{i} \boldsymbol{D}_{pure} \psi \, d^3x \; + \; \int E^i \, \mathcal{D}_{pure} \, A^i_{phys} \, d^3x \\ &= \boldsymbol{P}^{q\prime\prime} \; + \; \boldsymbol{P}^{g\prime\prime}, \end{aligned} \tag{9}$$

with the definition of the covariant derivatives, $\boldsymbol{D}_{pure} = \nabla - ig\boldsymbol{A}_{pure}$ and $\mathcal{D}_{pure} = \nabla - i\,g\,[\boldsymbol{A}_{pure}, \cdot\,]$. This is apparently different from the standardly known decomposition given by

$$\begin{aligned} \boldsymbol{P}_{QCD} &= \int \psi^\dagger \frac{1}{i} \boldsymbol{D} \, \psi \, d^3x \; + \; \int \boldsymbol{E} \times \boldsymbol{B} \, d^3x, \\ &= \boldsymbol{P}^q \; + \; \boldsymbol{P}^g, \end{aligned} \tag{10}$$

with $\boldsymbol{D} \equiv \nabla - i\,g\,\boldsymbol{A}$ being the convariant derivative containing the full gluon field $\boldsymbol{A}$. Based on the decomposition (9), they argue that the standard picture of the nucleon momentum partition within the framework of the perturbative QCD is drastically modified, thereby being led to a surprising conclusion that the gluon carries only about 1/5 of the total nucleon momentum in the asymptotic limit $Q^2 \to \infty$ [13], in sharp contrast to the standardly believed value 1/2. The conflict appears to stem from the fact that the decomposition of the total momentum into the quark and gluon parts is not unique even if the gauge invariance is imposed. The same problem turns out to occur also in the decomposition of the nucleon spin.

The purpose of the present paper is to clarify the relation between the known decompositions of the nucleon spin, and to show how they are related and in what respect they are critically different. We will show that the gauge-invariance requirement alone does not allow unique decomposition of the nucleon spin. As can be anticipated, the ambiguity originates from the quark-gluon interaction, which cannot simply be separated from the others for a strongly-coupled gauge system. It is shown that there exist two complete decompositions of the nucleon spin, i.e. the decomposition into the quark spin, the quark orbital angular momentum, the gluon spin, and the gluon orbital angular momentum, each of which is separately gauge invariant. One is the decomposition proposed by Chen et al., while the other is a new decomposition proposed in this paper. Which of these two decompositions is physically more preferable will be discussed from the standpoint of observability.

II. QED CASE

To make the essential physics as clear as possible, we start our analysis with the gauge-invariant decomposition of the total linear momentum in an interacting electron and photon system. Here, we rederive the decomposition of Cheng et al. in a slightly different manner as they did, since it is expected to clarify the physics meant by their decomposition. The starting point is the familiar gauge-invariant decomposition of the total linear momentum :

$$\begin{aligned} \boldsymbol{P}_{QED} \;&=\; \int \psi^\dagger \, \frac{1}{i} \, \boldsymbol{D} \, \psi \, d^3 x \;+\; \int \boldsymbol{E} \times \boldsymbol{B} \, d^3 x \\ &=\; \boldsymbol{P}^e \;+\; \boldsymbol{P}^\gamma, \end{aligned} \tag{11}$$

with $\boldsymbol{D} = \nabla - i \, e \, \boldsymbol{A}$. Similarly as Chen et al. did, we introduce a decomposition of the vector potential $\boldsymbol{A}$ into the longitudinal and transverse parts as

$$\boldsymbol{A} \;=\; \boldsymbol{A}_\parallel \;+\; \boldsymbol{A}_\perp, \tag{12}$$

with the conditions

$$\nabla \cdot \boldsymbol{A}_\perp \;=\; 0, \tag{13}$$

and

$$\nabla \times \boldsymbol{A}_\parallel \;=\; 0. \tag{14}$$

Here, we adopt the notation $\boldsymbol{A}_\parallel$ and $\boldsymbol{A}_\perp$ for the longitudinal and transverse components, since it is a more familiar notation. (The above decomposition is known to be unique, once the coordinate system is fixed [14]-[16].) The gauge transformation property of the relevant fields are given (in the natural unit) by

$$A^0 \;\longrightarrow\; A^{0\prime} \;=\; A^0 \;-\; \dot{\Lambda}(x), \tag{15}$$

$$\boldsymbol{A}_\parallel \;\longrightarrow\; \boldsymbol{A}_\parallel^\prime \;=\; \boldsymbol{A}_\parallel \;-\; \nabla \, \Lambda(x), \tag{16}$$

$$\boldsymbol{A}_\perp \;\longrightarrow\; \boldsymbol{A}_\perp^\prime \;=\; \boldsymbol{A}_\perp. \tag{17}$$

In correspondence with the above decomposition of $\boldsymbol{A}$, the electric field $\boldsymbol{E}$ can also be decomposed into the longitudinal and transverse parts as

$$\boldsymbol{E} \;=\; \boldsymbol{E}_\parallel \;+\; \boldsymbol{E}_\perp, \tag{18}$$

where

$$\boldsymbol{E}_\parallel \;=\; -\, \nabla A^0 \;-\; \dot{\boldsymbol{A}}_\parallel, \tag{19}$$

$$\boldsymbol{E}_\perp \;=\; -\, \dot{\boldsymbol{A}}_\perp. \tag{20}$$

On the other hand, only the transverse part of $\boldsymbol{A}$ contributes to the magnetic field $\boldsymbol{B}$, since

$$\boldsymbol{B} \;=\; \nabla \times \boldsymbol{A} \;=\; \nabla \times \boldsymbol{A}_{\perp}. \tag{21}$$

It is important to recognize that each part of $\boldsymbol{E}$, i.e. either of $\boldsymbol{E}_{\parallel}$ or $\boldsymbol{E}_{\perp}$, is separately invariant under the gauge transformations (16),(17). This means that the photon momentum $\boldsymbol{P}^{\gamma}$ in (11) can be gauge-invariantly decomposed into two parts as [14]-[16]

$$\boldsymbol{P}^{\gamma} \;=\; \boldsymbol{P}^{\gamma}_{long} \;+\; \boldsymbol{P}^{\gamma}_{trans}, \tag{22}$$

with

$$\boldsymbol{P}^{\gamma}_{long} \;=\; \int \boldsymbol{E}_{\parallel} \times \boldsymbol{B} \, d^3x \;=\; \int \boldsymbol{E}_{\parallel} \times (\nabla \times \boldsymbol{A}_{\perp}) \, d^3x, \tag{23}$$

$$\boldsymbol{P}^{\gamma}_{trans} \;=\; \int \boldsymbol{E}_{\perp} \times \boldsymbol{B} \, d^3x \;=\; \int \boldsymbol{E}_{\perp} \times (\nabla \times \boldsymbol{A}_{\perp}) \, d^3x. \tag{24}$$

Using the transverse condition $\nabla \cdot \boldsymbol{A}_{\perp} = 0$, $\boldsymbol{P}^{\gamma}_{trans}$ can readily be transformed into the form

$$\boldsymbol{P}^{\gamma}_{trans} \;=\; \int E^j_{\perp} \, \nabla \, A^j_{\perp} \, d^3x. \tag{25}$$

On the other hand, after partial integration and dropping the surface term, $\boldsymbol{P}^{long}_{\gamma}$ can be written as

$$\boldsymbol{P}^{\gamma}_{long} \;=\; \int (\nabla E^j_{\parallel}) \, A^j_{\perp} \, d^3x \;+\; \int (\nabla \cdot \boldsymbol{E}_{\parallel}) \, \boldsymbol{A}_{\perp} \, d^3x. \tag{26}$$

Here, by using the transverse condition $\nabla \cdot \boldsymbol{A}_{\perp} = 0$, the first term can be shown to vanish after partial integration. (The proof is elementary if one notices the fact that $\boldsymbol{E}_{\parallel}$ is represented as a gradient of some scalar function. This is self-evident in the Coulomb gauge in which $\boldsymbol{A}_{\parallel} = 0$ so that $\boldsymbol{E}_{\parallel} = -\nabla A^0$. However, it also holds in arbitrary gauge connected with the Coulomb gauge through general gauge transformation.) Then, by using the Gauss law $\nabla \cdot \boldsymbol{E} = \nabla \cdot \boldsymbol{E}_{\parallel} = \rho$, $\boldsymbol{P}^{\gamma}_{long}$ can be written as

$$\boldsymbol{P}^{\gamma}_{long} \;=\; \int \rho \, \boldsymbol{A}_{\perp} \, d^3x. \tag{27}$$

Since the charge density is given as $\rho = e \, \psi^{\dagger} \psi$ by the electron field, $\boldsymbol{P}^{long}_{\gamma}$ can also be expressed as

$$\boldsymbol{P}^{\gamma}_{long} \;=\; \int \psi^{\dagger} \, e \, \boldsymbol{A}_{\perp} \, \psi \, d^3x. \tag{28}$$

To sum up, the total momentum of an interacting electron and photon system is given by

$$\begin{aligned}\boldsymbol{P}_{QED} \;&=\; \int \psi^\dagger \,(\boldsymbol{p} - e\boldsymbol{A})\,\psi\, d^3x \\ &+\; \int \psi^\dagger \, e\, \boldsymbol{A}_\perp \,\psi\, d^3x \;+\; \int E^j_\perp \,\nabla\, A^j_\perp \, d^3x, \end{aligned} \tag{29}$$

where $\boldsymbol{p}$ is the canonical momentum operator given by $\boldsymbol{p} = \frac{1}{i}\nabla$. If one combines the 1st and 2nd terms of the above equation, one obtains

$$\begin{aligned}\boldsymbol{P}_{QED} \;&=\; \int \psi^\dagger \,(\boldsymbol{p} - e\,\boldsymbol{A}_\parallel)\,\psi\, d^3x \;+\; \int E^j_\perp \,\nabla\, A^j_\perp \, d^3x, \\ &\equiv\; \boldsymbol{P}^{e\prime\prime} \;+\; \boldsymbol{P}^{\gamma\prime\prime}, \end{aligned} \tag{30}$$

which precisely corresponds to the decomposition advocated by Chen et al. in the QED case. (Note that this is a gauge-invariant decomposition.)

On the other hand, of one includes the 2nd term of (29) into the photon part, one obtains

$$\boldsymbol{P}_{QED} \;=\; \boldsymbol{P}^e \;+\; \boldsymbol{P}^\gamma, \tag{31}$$

where

$$\boldsymbol{P}^e \;=\; \int \psi^\dagger \,(\boldsymbol{p} - e\,\boldsymbol{A})\,\psi\, d^3x, \tag{32}$$

$$\boldsymbol{P}^\gamma \;=\; \int E^j_\perp \,\nabla\, A^j_\perp \, d^3x \;+\; \int \rho\, \boldsymbol{A}_\perp \, d^3x. \tag{33}$$

One must then conclude that there exist two gauge-invariant decompositions, i.e. (30) and (31), of the total momentum of the coupled electron-photon system. This arbitrariness of the decomposition arises, since each term of (29) is separately gauge invariant, so that the gauge-invariance requirement alone cannot answer the question which of the electron or photon part the 2nd term of (29) should be incorporated into. Chen et al. advocated to include it into the electron momentum part. This however appears to contradict the standard understanding of the electrodynamics. As already emphasized by Ji [17], the *kinetic* or *dynamical* momentum of a charged particle is $\boldsymbol{\Pi} = \boldsymbol{p} - q\,\boldsymbol{A}$ not $\boldsymbol{p} - q\,\boldsymbol{A}_\parallel$. (By the term *kinetic* or *dynamical* momentum, we mean the momentum accompanying the mass flow of a moving charged particle.) This seems clear from the fact that $\boldsymbol{\Pi}$ appears in the quantum-mechanical version of the Lorentz force equation controlling motion of a charged particle. (See, for example, [18].) The decomposition (31) does not suffer from this problem, in the sense that the *dynamical* momentum legitimately appears in the electron part. In

return for this advantage, however, the photon part is forced to contain an extra piece, i.e. $\int \psi^\dagger \, e \, \boldsymbol{A}_\perp \, \psi \, d^3x \; = \; \int \rho \, \boldsymbol{A}_\perp \, d^3x$. A key question is therefore the physical meaning of this extra piece in the photon momentum $\boldsymbol{P}^\gamma$. Remember that it originates from $\boldsymbol{P}^\gamma_{long}$ given by (23). It therefore seems clear that this momentum is associated with the longitudinal electric field created by the electrons. To back up this interpretation further, let us consider the case in which the matter field is a collection of moving charged particles with the charges q_i, which indicates the replacement

$$e \, \psi^\dagger(x) \, \psi(x) \;\longrightarrow\; \sum_i q_i \, \delta(\boldsymbol{x} - \boldsymbol{r}_i). \tag{34}$$

In this case, we find that

$$\boldsymbol{P}^\gamma_{long} \;=\; \int \psi^\dagger(x) \, e \, \boldsymbol{A}_\perp \, \psi(x) \, d^3x \;\longrightarrow\; \sum_i q_i \, \boldsymbol{A}_\perp(\boldsymbol{r}_i). \tag{35}$$

One then confirms that $q_i \, \boldsymbol{A}_\perp(\boldsymbol{r}_i)$ is the momentum associated with the longitudinal (photon) field created by the charged particle i. To borrow Konopinski's words [19], one may say, just as $q \, \phi$ serves as a "store" of field energy, $q \, \boldsymbol{A}_\perp$ measures a store of field momentum available to the charge's motion. He even advocated a viewpoint : Those who prefer to call $q \, \phi$ a potential energy might adopt the name "potential momentum" for $q \, \boldsymbol{A}_\perp$. In any case, we now clearly recognize the fact that the existence of two gauge-invariant decompositions of the total momentum in QED is connected with the arbitrariness that the potential momentum can be assigned to either of a part of the electron momentum or as a part of the photon momentum. Obviously, it is a problem inherent in the strongly coupled gauge system, in which the interaction between the constituents cannot be separated in a trivial way.

Next, we turn to more interesting case of angular-momentum decomposition in QED. We start with the familiar gauge-invariant decomposition given as

$$\boldsymbol{J}_{QED} \;=\; \int \psi^\dagger \, \frac{1}{2} \, \boldsymbol{\Sigma} \, \psi \, d^3x \;+\; \int \psi^\dagger \, \boldsymbol{x} \times (\boldsymbol{p} - e \, \boldsymbol{A}) \, \psi \, d^3x \;+\; \boldsymbol{J}^\gamma, \tag{36}$$

with

$$\boldsymbol{J}^\gamma \;=\; \int \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B}) \, d^3x. \tag{37}$$

We decompose $\boldsymbol{J}^\gamma$ into two parts as

$$\boldsymbol{J}^\gamma \;=\; \boldsymbol{J}^\gamma_{long} \;+\; \boldsymbol{J}^\gamma_{trans}, \tag{38}$$

with

$$\boldsymbol{J}^{\gamma}_{long} \;=\; \int \boldsymbol{x} \times (\boldsymbol{E}_{\parallel} \times \boldsymbol{B})\, d^3x \;=\; \int \boldsymbol{x} \times [\boldsymbol{E}_{\parallel} \times (\nabla \times \boldsymbol{A}_{\perp})]\, d^3x, \tag{39}$$

$$\boldsymbol{J}^{\gamma}_{trans} \;=\; \int \boldsymbol{x} \times (\boldsymbol{E}_{\perp} \times \boldsymbol{B})\, d^3x \;=\; \int \boldsymbol{x} \times [\boldsymbol{E}_{\perp} \times (\nabla \times \boldsymbol{A}_{\perp})]\, d^3x. \tag{40}$$

After straightforward algebra, i.e. partial integration with surface term dropped, $\boldsymbol{J}^{\gamma}_{long}$ can be rewritten in the following form :

$$\boldsymbol{J}^{\gamma}_{long} \;=\; \int [\, E^j_{\parallel}\,(\boldsymbol{x} \times \nabla)\, A^j_{\perp} \;+\; (\boldsymbol{x} \times \boldsymbol{A}_{\perp})\, \nabla \cdot \boldsymbol{E}_{\parallel} \;+\; \boldsymbol{E}_{\parallel} \times \boldsymbol{A}_{\perp}\,]\, d^3x. \tag{41}$$

Using one of the Maxwell equations

$$\nabla \cdot \boldsymbol{E}_{\parallel} \;=\; \rho \;=\; e\, \psi^{\dagger}\, \psi, \tag{42}$$

we thus obtain

$$\boldsymbol{J}^{\gamma}_{long} \;=\; \int \psi^{\dagger}\, \boldsymbol{x} \times\, e\, \boldsymbol{A}_{\perp}\, \psi\, d^3x \;+\; \int [\, E^j_{\parallel}\,(\boldsymbol{x} \times \nabla)\, A^j_{\perp} \;+\; \boldsymbol{E}_{\parallel} \times \boldsymbol{A}_{\perp}\,]\, d^3x. \tag{43}$$

It can be shown that the 2nd term of the above equation identically vanishes, i.e.

$$\int [\, E^j_{\parallel}\,(\boldsymbol{x} \times \nabla)\, A^j_{\perp} \;+\; \boldsymbol{E}_{\parallel} \times \boldsymbol{A}_{\perp}\,]\, d^3x \;=\; 0. \tag{44}$$

The proof is easiest in the Coulomb gauge in which $\boldsymbol{A}_{\parallel} = 0$ and $\boldsymbol{E}_{\parallel} = -\,\nabla\, A^0$, but the result itself is correct in arbitrary gauge in which $\boldsymbol{E}_{\parallel}$ is expressed as a gradient of some scalar function. As a consequence, we find that

$$\boldsymbol{J}^{\gamma}_{long} \;=\; \int \psi^{\dagger}\, \boldsymbol{x} \times\, e\, \boldsymbol{A}_{\perp}\, \psi\, d^3x. \tag{45}$$

On the other hand, $\boldsymbol{J}^{\gamma}_{trans}$ can be rewritten as

$$\boldsymbol{J}^{\gamma}_{trans} \;=\; \int [\, \boldsymbol{E}_{\perp} \times \boldsymbol{A}_{\perp} \;+\; E^j_{\perp}\,(\boldsymbol{x} \times \nabla)\, A^j_{\perp}\,]\, d^3x. \tag{46}$$

Here, use has been made of the relation $\nabla \cdot \boldsymbol{E}_{\perp} = 0$. To sum up, we obtain

$$\begin{aligned} \boldsymbol{J}^{\gamma} \;&\equiv\; \boldsymbol{J}^{\gamma}_{long} \;+\; \boldsymbol{J}^{\gamma}_{trans} \\ &=\; \int \psi^{\dagger}\, \boldsymbol{x} \times\, e\, \boldsymbol{A}_{\perp}\, \psi\, d^3x \;+\; \int [\, \boldsymbol{E}_{\perp} \times \boldsymbol{A}_{\perp} \;+\; E^j_{\perp}\,(\boldsymbol{x} \times \nabla)\, A^j_{\perp}\,]\, d^3x. \end{aligned} \tag{47}$$

Because of the relation (44), the above $\boldsymbol{J}^{\gamma}$ can equivalently be expressed as

$$\begin{aligned} \boldsymbol{J}^{\gamma} \;&=\; \int \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B})\, d^3x \\ &=\; \int \psi^{\dagger}\, \boldsymbol{x} \times\, e\, \boldsymbol{A}_{\perp}\, \psi\, d^3x \;+\; \int [\, \boldsymbol{E} \times \boldsymbol{A}_{\perp} \;+\; E^j\,(\boldsymbol{x} \times \nabla)\, A^j_{\perp}\,]\, d^3x. \end{aligned} \tag{48}$$

Since $\boldsymbol{E}$ and $\boldsymbol{A}_\perp$ are both gauge invariant, it is obvious that each term of the above equation is separately gauge invariant. In particular, the 2nd and 3rd terms of the above decomposition corresponds to the intrinsic spin and orbital angular momentum of a photon. (To be more precise, those of an isolated photon. See the discussion below.) It is widely believed that a gauge-invariant decomposition of thetotal photon angular momentum into the spin and orbital parts is impossible. This statement appears to need a slight modification. Such decomposition is not impossible, although it contains an extra piece inherent in a strongly coupled gauge system. The extra piece is

$$\boldsymbol{J}^\gamma_{long} \;=\; \int \boldsymbol{x} \times (\boldsymbol{E}_\parallel \times \boldsymbol{B})\, d^3x \;=\; \int \psi^\dagger \, \boldsymbol{x} \times \, e\, \boldsymbol{A}_\perp \, \psi \, d^3x \;=\; \int \rho \, \boldsymbol{x} \times \boldsymbol{A}_\perp \, d^3x, \tag{49}$$

which might well be called the "potential angular momentum" as a generalization of Konopinski's potential momentum. A new gauge-invariant decomposition of $\boldsymbol{J}_{QED}$ by Chen et al. is obtained by including this term into the electron orbital angular-momentum part, which leads to

$$\begin{aligned}
\boldsymbol{J}_{QED} \;&=\; \int \psi^\dagger \, \frac{1}{2} \boldsymbol{\Sigma} \, \psi \, d^3x \;+\; \int \psi^\dagger \, \boldsymbol{x} \times (\boldsymbol{p} - e\, \boldsymbol{A}_\parallel) \, \psi \, d^3x \\
&\quad + \int \boldsymbol{E} \times \boldsymbol{A}_\perp \, d^3x \;+\; \int E^j \, (\boldsymbol{x} \times \nabla) \, A^j_\perp \, d^3x \\
&\equiv\; \boldsymbol{S}^e \;+\; \boldsymbol{L}^{e\prime\prime} \;+\; \boldsymbol{S}^{\gamma\prime\prime} \;+\; \boldsymbol{L}^{\gamma\prime\prime}.
\end{aligned} \tag{50}$$

However, this is not the only possibility. Another gauge-invariant decomposition is obtained by including the term $\boldsymbol{J}^\gamma_{long}$ into the photon orbital angular-momentum part :

$$\boldsymbol{J}_{QED} \;=\; \boldsymbol{S}^e \;+\; \boldsymbol{L}^e \;+\; \boldsymbol{S}^\gamma \;+\; \boldsymbol{L}^\gamma, \tag{51}$$

with

$$\boldsymbol{L}^e \;=\; \int \psi^\dagger \, \boldsymbol{x} \times (\boldsymbol{p} - e\, \boldsymbol{A}) \, \psi \, d^3x, \tag{52}$$

$$\boldsymbol{S}^\gamma \;=\; \int \boldsymbol{E} \times \boldsymbol{A}_\perp \, d^3x, \tag{53}$$

$$\boldsymbol{L}^\gamma \;=\; \int E^j \, (\boldsymbol{x} \times \nabla) \, A^j_\perp \, d^3x \;+\; \int \rho \, \boldsymbol{x} \times \boldsymbol{A}_\perp \, d^3x. \tag{54}$$

Here, $\boldsymbol{S}^\gamma = \boldsymbol{S}^{\gamma\prime\prime}$. (We could have included the term $\boldsymbol{J}^\gamma_{long}$ into the photon spin part in the new decomposition as well. However, we believe that our choice is natural, since the term $\rho \, \boldsymbol{x} \times \boldsymbol{A}_\perp$ takes the form of a vector product of the coordinate vector $\boldsymbol{x}$ and the potential

momentum $\rho \boldsymbol{A}_\perp$ *a la* Konopinski.) By construction, the sum of $\boldsymbol{S}^\gamma$ and $\boldsymbol{L}^\gamma$ just reduces to the total photon angular momentum $\boldsymbol{J}^\gamma = \int \boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B}) \, d^3x$, up to a surface term. However, note that (51) realizes a gauge-invariant decomposition of $\boldsymbol{J}^\gamma$ into the spin and orbital parts. Again, we are led to the conclusion that the gauge invariance alone does not allow unique decomposition of the total angular momentum of the strongly coupled electron-photon system. We prefer the decomposition (51), since the *dynamical* orbital angular momentum appears legitimately in the electron part. In spite of this standard view, which decomposition is physically appealing must after all be judged from the standpoint of observability. We shall come back to this point when discussing the nucleon spin problem in QCD in the next section.

III. QCD CASE

Now, we turn to the QCD case of our primary concern, In this case, some additional complication arises due to the non-Abelian nature of the relevant gauge theory. Fortunately, as long as the problem in question is concerned, the essential physics does not seem to change from the QED case, as we shall see below. Let us start again with the most popular gauge-invariant decomposition of the nucleon spin :

$$\begin{aligned} \boldsymbol{J}_{QCD} &= \int \psi^\dagger \, \frac{1}{2} \boldsymbol{\Sigma} \, \psi \, d^3x \; + \; \int \psi^\dagger \, \boldsymbol{x} \times \frac{1}{i} \boldsymbol{D} \, \psi \, d^3x \; + \; \int \boldsymbol{x} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a) \, d^3x \\ &= \boldsymbol{S}^q \; + \; \boldsymbol{L}^q \; + \; \boldsymbol{J}^g, \end{aligned} \tag{55}$$

where $\boldsymbol{E} = \boldsymbol{E}^a \, T^a$ and $\boldsymbol{B} = \boldsymbol{B}^a \, T^a$ with T^a being the color SU(3) generators. We first notice that, by using the equation

$$\boldsymbol{B}^a \;=\; \nabla \times \boldsymbol{A}^a \;+\; \frac{1}{2} \, g \, f_{abc} \, \boldsymbol{A}^b \times \boldsymbol{A}^c, \tag{56}$$

for the color magnetic field, we can prove the following identity :

$$(\boldsymbol{E}^a \times \boldsymbol{B}^a)^i \;=\; E^{aj} \, \nabla^i \, A^{aj} \;+\; (\mathcal{D} \cdot \boldsymbol{E})^a \, A^{ai} \;-\; \nabla^j \, (E^{aj} \, A^{ai}). \tag{57}$$

Here

$$(\mathcal{D} \cdot \boldsymbol{E})^a \;\equiv\; (\nabla \cdot \boldsymbol{E} \;-\; i \, g \, [\boldsymbol{A}, \boldsymbol{E}] \,)^a \;=\; \nabla \cdot \boldsymbol{E}^a \;+\; g \, f_{abc} \, \boldsymbol{A}^b \cdot \boldsymbol{E}^c. \tag{58}$$

Using the above identity, we thus obtain

$$\int \boldsymbol{x} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a) \;=\; \int E^{aj}\,(\boldsymbol{x} \times \nabla)\,A^{aj}\,d^3x \;+\; \int (\mathcal{D} \cdot \boldsymbol{E})^a\,\boldsymbol{x} \times \boldsymbol{A}^a\,d^3x$$
$$+\; \int \boldsymbol{E}^a \times \boldsymbol{A}^a\,d^3x \;-\; \int \nabla^j\,[E^{aj}\,(\boldsymbol{x} \times \boldsymbol{A})\,]\,d^3x. \tag{59}$$

Next, using the Gauss law

$$(\mathcal{D} \cdot \boldsymbol{E})^a \;=\; \rho^a \;=\; g\,\psi^\dagger\,T^a\,\psi, \tag{60}$$

and simply dropping the last surface term in (59), we obtain

$$\int \boldsymbol{x} \times (\boldsymbol{E}^q \times \boldsymbol{B}^a)\,d^3x \;=\; \int g\,\psi^\dagger\,\boldsymbol{x} \times \boldsymbol{A}\,\psi\,d^3x$$
$$+\; \int \boldsymbol{E}^a \times \boldsymbol{B}^a\,d^3x \;+\; \int E^{aj}\,(\boldsymbol{x} \times \nabla)\,A^{aj}\,d^3x. \tag{61}$$

Combining this with the quark parts, we are then led to

$$\begin{aligned} \boldsymbol{J}_{QCD} \;&=\; \int \psi^\dagger\,\frac{1}{2}\,\boldsymbol{\Sigma}\,\psi\,d^3x \;+\; \int \psi^\dagger\,\boldsymbol{x} \times \frac{1}{i}\,\nabla\,\psi\,d^3x \\ &+\; \int \boldsymbol{E}^a \times \boldsymbol{B}^a\,d^3x \;+\; \int E^{aj}\,(\boldsymbol{x} \times \nabla)\,A^{aj}\,d^3x \\ &=\; \boldsymbol{S}^q \;+\; \boldsymbol{L}^{q\prime} \;+\; \boldsymbol{S}^{g\prime} \;+\; \boldsymbol{L}^{g\prime}, \end{aligned} \tag{62}$$

which is nothing but the Jaffe-Manohar decomposition of the nucleon spin. As already pointed out, an unpleasant feature of this decomposition is that each term is not separately gauge-invariant except for the intrinsic quark spin part.

Generalizing the longitudinal and transverse decomposition of the photon field in QED, Chen et al. proposed a decomposition of the gluon field into two parts as $A^\mu = A^\mu_{pure} + A^\mu_{phys}$, with A^μ_{pure} a pure-gauge term transforming in the same way as the full A^μ does, and always giving null field strength (i.e. $F^{\mu\nu}_{pure} \equiv \partial^\mu A^\mu_{pure} - \partial^\nu A^\mu_{pure} + i\,g\,[A^\mu_{pure}, A^\nu_{pure}] = 0$), and A^μ_{phys} a physical part of A^μ transforming in the same manner as $F^{\mu\nu}$ does, i.e. covariantly. They argue that this decomposition is basically unique, once $\boldsymbol{A}_{phys}$ is chosen to satisfy either of the defining equations :

$$[\,\boldsymbol{A}_{phys},\,\boldsymbol{E}\,] \;=\; 0, \tag{63}$$

or

$$\mathcal{D}_{pure} \cdot \boldsymbol{A}_{phys} \;=\; 0, \tag{64}$$

with $\mathcal{D}^\mu_{pure} \equiv \partial^\mu - i\,g\,[A^\mu_{pure}, \cdot\,]$.

To be fair, we should mention here the existence of several criticism to this decomposition [17],[20],[21]. (See also the objections to these criticisms [22] -[24].) For instance, the Lorentz-frame-dependent as well as nonlocal nature of this decomposition was criticized by Ji. In our opinion, this noncovariant feature of the treatment is not an essential trouble of the decomposition. In fact, the physical significance of the corresponding decomposition in the QED case was well established by now. (See, for instance, the textbook [16].) What is not still completely confident to us is the uniqueness of the decomposition in the case of non-Abelian gauge theory. Another question is whether the frequently used manipulation, i.e. the neglect of surface term, is justified also in the case of QCD. The answer to this question may not be trivial, because we know the existence of Gribov ambiguity for the nonperturbative non-Abelian gauge field configuration, and because the gluon field configuration with nontrivial topology might play some unexpected role in the nucleon structure. These difficult problems would be answered only after one can accomplish the proper (nonperturbative) quantization of gauge field in the canonical form or, using the Fadeev-Popov-trick, in the path integral formulation, and they are beyond the scope of the present investigation. Nonetheless, one should keep in mind the fact that there still remains a lot of questions to be answered on the above decomposition of the nonabelian gauge field.

In the following discussion, we assume that this decomposition is unique. Then, another decomposition of the nucleon spin can be obtained by the following procedure. That is, in the 4th surface term of (59), we drop only the part containing the physical part of $\boldsymbol{A}$, which is equivalent to retaining the surface term given by

$$- \int \nabla^j \left[E^{aj} \left(\boldsymbol{x} \times \boldsymbol{A}^a_{pure} \right) \right] d^3x, \tag{65}$$

in (59). Using the relation

$$\nabla \cdot \boldsymbol{E}^a \; + \; g \, f_{abc} \, \boldsymbol{A}^b_{pure} \cdot \boldsymbol{E}^c \; = \; \rho^a, \tag{66}$$

which follows from the standard Gauss law

$$(\mathcal{D} \cdot \boldsymbol{E})^a \; \equiv \; \nabla \cdot \boldsymbol{E}^a \; + \; g \, f_{abc} \, \boldsymbol{A}^b \cdot \boldsymbol{E}^c \; = \; \rho^a, \tag{67}$$

combined with the condition

$$[\, \boldsymbol{A}_{phys}, \, \boldsymbol{E} \,] \; = \; \boldsymbol{A}_{phys} \cdot \boldsymbol{E} \; - \; \boldsymbol{E} \cdot \boldsymbol{A}_{phys} \; = \; 0, \tag{68}$$

we can then prove the following identity :

$$\int \nabla^j \left[E^{aj} \left(\boldsymbol{x} \times \boldsymbol{A}^a_{phys} \right) \right] d^3x \;=\; \int g\, \psi^\dagger\, \boldsymbol{x} \times \boldsymbol{A}_{pure}\, \psi\, d^3x \\ +\; \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{pure}\, d^3x \;+\; \int E^{aj} \left(\boldsymbol{x} \times \nabla \right) A^{aj}_{pure}\, d^3x. \quad (69)$$

Here, we have used the relation $\rho^a \left(\boldsymbol{x} \times \boldsymbol{A}^a_{pure} \right) = g\, \psi^\dagger\, \boldsymbol{x} \times \boldsymbol{A}_{pure}\, \psi$. Using it, we get

$$\int \boldsymbol{x} \times \left(\boldsymbol{E}^a \times \boldsymbol{B}^a \right) d^3x \;=\; \int g\, \psi^\dagger\, \boldsymbol{x} \times \boldsymbol{A}_{phys}\, \psi\, d^3x \\ +\; \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys}\, d^3x \;+\; \int E^{aj} \left(\boldsymbol{x} \times \nabla \right) A^{aj}_{phys}\, d^3x, \quad (70)$$

which is a generalization of (48) in the QED case. Note that each term of this decomposition of $\boldsymbol{J}^g$ is separately gauge invariant. This fact can easily be convinced, if one remembers the covariant transformation property of $\boldsymbol{A}_{phys}$, i.e.

$$\boldsymbol{A}_{phys} \;\longrightarrow\; \boldsymbol{A}'_{phys} \;=\; U(x)\, \boldsymbol{A}_{phys}\, U^\dagger(x). \quad (71)$$

Since $\boldsymbol{A} - \boldsymbol{A}_{phys} = \boldsymbol{A}_{pure}$, the nucleon spin decomposition of Chen et al. is obtained by including the 1st term of (59) into the quark orbital angular-momentum part :

$$\begin{aligned} \boldsymbol{J}_{QCD} \;&=\; \int \psi^\dagger\, \frac{1}{2}\, \boldsymbol{\Sigma}\, \psi\, d^3x \;+\; \int \psi^\dagger\, \boldsymbol{x} \times \left(\boldsymbol{p} - g\, \boldsymbol{A}_{pure} \right) \psi\, d^3x \\ &+\; \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys}\, d^3x \;+\; \int E^{aj} \left(\boldsymbol{x} \times \nabla \right) A^{aj}_{phys}\, d^3x \\ &=\; \boldsymbol{S}^q \;+\; \boldsymbol{L}^{q\prime\prime} \;+\; \boldsymbol{S}^{g\prime\prime} \;+\; \boldsymbol{L}^{g\prime\prime}. \end{aligned} \quad (72)$$

A noteworthy fact, which was pointed out by Chen et al., is that, in a particular gauge $[\boldsymbol{A}, \boldsymbol{E}] = 0$, i.e. in what they call the generalized Coulomb gauge (together with possible supplementary conditions to completely fix the gauge), the decomposition (72) is reduced to the gauge-variant decomposition of Jaffe and Manohar. Since each term of the decomposition (72) is separately gauge-invariant, this already implies that the numerical value of each term obtained from the decomposition (72) is nothing different from the answer of the Jaffe-Manohar decomposition.

However, one should remember the fact that the term $g\, \psi^\dagger\, \boldsymbol{x} \times \boldsymbol{A}_{phys}\, \psi$, that can also be expressed as $\rho^a \left(\boldsymbol{x} \times \boldsymbol{A}^a_{phys} \right)$, is a correspondent of $\rho \left(\boldsymbol{x} \times \boldsymbol{A}_\perp \right)$ in the QED case, which has been interpreted as a store of angular momentum generated by the charge's motion. To include this term into the quark angular momentum would not be justified in view of

our standard understanding of the electrodynamics, in which the *kinematical* or *dynamical* momentum of a charged particle is $\mathbf{\Pi} = \boldsymbol{p} - g\,\boldsymbol{A}$ not $\boldsymbol{p} - g\,\boldsymbol{A}_{\parallel}$.

We therefore propose to include this term into the orbital angular momentum carried by the gluon field. This leads to a new decomposition of the nucleon spin given as

$$\boldsymbol{J}_{QCD} \;=\; \boldsymbol{S}^q \;+\; \boldsymbol{L}^q \;+\; \boldsymbol{S}^g \;+\; \boldsymbol{L}^g, \tag{73}$$

where

$$\boldsymbol{S}^q \;=\; \int \psi^\dagger \, \frac{1}{2} \, \mathbf{\Sigma} \, \psi \, d^3x, \tag{74}$$

$$\boldsymbol{L}^q \;=\; \int \psi^\dagger \, \boldsymbol{x} \times (\boldsymbol{p} - g\,\boldsymbol{A}) \, \psi \, d^3x, \tag{75}$$

$$\boldsymbol{S}^g \;=\; \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3x, \tag{76}$$

$$\boldsymbol{L}^g \;=\; \int E^{aj} \, (\boldsymbol{x} \times \nabla) \, A^{aj}_{phys} \, d^3x \;+\; \int \rho^a \, (\boldsymbol{x} \times \boldsymbol{A}^a_{phys}) \, d^3x. \tag{77}$$

We emphasize once again that each piece of this decomposition is separately gauge-invariant.

After all, we now have two gauge-invariant decompositions of the nucleon spin, i.e. (72) and (73), both of which enables the separation of the gluon total angular momentum into the spin and orbital parts. Clearly, the gauge principle alone cannot judge which decomposition is preferable. We shall now develop an argument in favor of the latter decomposition. First, as repeatedly emphasized, the knowledge of the standard electrodynamics tells us that the orbital angular momentum accompanying the mass flow of the charged particle motion is the *dynamical* orbital angular momentum $\boldsymbol{x} \times \mathbf{\Pi} = \boldsymbol{x} \times (\boldsymbol{p} - g\,\boldsymbol{A})$ not $\boldsymbol{x} \times (\boldsymbol{p} - g\boldsymbol{A}_{\parallel})$. (The latter can be thought of as a nontrivial generalization of the canonical orbital angular momentum $\boldsymbol{x} \times \boldsymbol{p}$.) Notice that the quark part of (73) is nothing different from the Ji decomposition. It is a widely known fact that the total angular momentum $\boldsymbol{J}^q \equiv \boldsymbol{S}^q + \boldsymbol{L}^q$ carried by the quark field in the nucleon can in principle be measured through the analysis of unpolarized generalized parton distributions $E^q(x, \xi, t)$ [25]. Since the intrinsic quark spin part $\boldsymbol{S}^q$ is already well-known from the polarized deep-inelastic scatterings [1] -[4], the orbital angular momentum of quarks as defined by (73) is clearly a measurable quantity, although somewhat indirectly.

What about the gluon part, then ? Certainly, an experimental decomposition of the gluon angular momentum is much more delicate than the quark part. At this point, we think it useful to remember the investigation by Bashinsky and Jaffe [26]. They invented

a method of constructing gauge-invariant quark and gluon distributions describing abstract QCD observables and apply it for analyzing angular momentum of the nucleon. In addition to the known quark and gluon polarized distribution functions, they gave a definition of gauge-invariant distributions for quark and gluon orbital angular momenta. They further argue that the 1st moments of these distribution functions should give the total quark/gluon spin/orbital momenta in the nucleon in the infinite momentum frame, and that the sum of these first moments satisfies the angular-momentum sum rule of the nucleon. Very interestingly, although each term of this 1st moment sum rule is separately gauge-invariant, it was shown to reduce to the Jaffe-Manohar decomposition of the nucleon spin in a particular gauge, i.e. the light-cone gauge $A^+ = 0$ and in the infinite momentum frame. As pointed out in their paper, this implies that the gluon spin part of the Jaffe-Manohar decomposition can be measured through the polarized deep-inelastic-scattering processes, as is naively expected. (Also noteworthy here is the following observation. As we have already pointed out, the gluon spin part in Chen et al's decomposition was claimed to reduce that of the Jaffe-Monohar in a particular gauge, i.e. in what-they-call the generalized Coulomb gauge. Note also that the gluon spin part is common in Chen et al's decomposition and our present decomposition. These observations altogether strongly indicate that at least the gluon spin part is common in all four decompositions, i.e. the Jaffe-Manohar decomposition, the Bashinsky-Jaffe one, the decomposition by Chen et al., and our present one, except for unphysical degrees of freedom of gauge transformation.)

However, Bashinsky and Jaffe could not offer any practical experimental means that can be used to measure the x-distributions of quark and gluon orbital angular momenta appearing in their defining equation. This may have some connection with the fact that the quark orbital angular momentum appearing in the Jaffe-Manohar decomposition is the *canonical* orbital angular momentum and not the *dynamical* orbital angular momentum. (The problem here is not the gauge-variant nature of the Jaffe-Manohar decomposition, since this decomposition can now be thought of as a gauge-fixed form of the gauge-invariant Bashinsky-Jaffe decomposition or that of the Chen et al's decomposition.) We have already indicated that the quark orbital angular momentum, which can be measured through the GPD analysis, is the *dynamical* orbital angular momentum $\boldsymbol{L}^q$ appearing in our new decomposition (73), or in the famous Ji decomposition, not $\boldsymbol{L}^{q\prime}$ appearing in the decomposition (72) of Chen et al. (This seems understandable if one remembers the following fact. We have already pointed

out that the momentum accompanying the mass flow of a charged particle is the *dynamical* momentum $\boldsymbol{\Pi} = \boldsymbol{p} - g\,\boldsymbol{A}$ containing the full gauge field and not $\boldsymbol{p} - g\,\boldsymbol{A}_{pure}$. Similarly, the angular momentum accompanying the mass flow is the *dynamical* angular momentum not the canonical angular momentum or its gauge-invariant generalization. Such flows of mass would in principle be detected through the coupling with the gravitational field. The appearance of the gravito-electric and gravito-magnetic form factors in Ji's nucleon spin sum rule would not be a mere coincidence in this sense.)

It is clear by now that the difference between these two types of decompositions is just concerned with the orbital angular momenta of quark and gluon. The relation between them is

$$\boldsymbol{L}^q \;+\; \boldsymbol{L}^g \;=\; \boldsymbol{L}^{q\prime\prime} \;+\; \boldsymbol{L}^{g\prime\prime}, \tag{78}$$

with

$$\begin{aligned} \boldsymbol{L}^g \;-\; \boldsymbol{L}^{g\prime\prime} \;&=\; -\,(\,\boldsymbol{L}^q \;-\; \boldsymbol{L}^{q\prime\prime}\,) \\ &=\; \int \rho^a\,(\boldsymbol{x} \times \boldsymbol{A}^a_{phys})\,d^3x \;\equiv\; \text{“potential angular momentum”}. \end{aligned} \tag{79}$$

Here, $\boldsymbol{L}^q$ and $\boldsymbol{L}^g$ are the quark and gluon orbital angular momenta in our new decomposition (73). On the other hand, $\boldsymbol{L}^{q\prime\prime}$ and $\boldsymbol{L}^{g\prime\prime}$ are the corresponding orbital angular momenta in the decomposition (72) of Chen et al. (The latter should be numerically equal to those of Jaffe and Manohar, in view of the fact that the latter can be thought of as a gauge-fixed form of the former.) The gluon orbital angular momentum $\boldsymbol{J}^g \;\equiv\; \boldsymbol{S}^g \,+\, \boldsymbol{L}^g$ defined in the decomposition (73), or more precisely the nucleon matrix element of $J^g_3 \;\equiv\; S^g_3 \,+\, L^g_3$, is expected to be measured through the GPD analysis, or at the least it can be extracted from the relation $\langle J^g_3 \rangle = 1/2 - \langle J^q_3 \rangle$. Here $\langle\ \rangle$ is an abbreviation of the appropriate nucleon matrix element. On the other hand, as our argument above strongly indicates, the nucleon matrix element of S^g_3 is essentially the same quantity as extracted from the polarized deep-inelastic-scattering measurements. (To make this statement more precise, we certainly need some additional works.) It means that $\langle L^g_3 \rangle$ is extracted from $\langle J^g_3 \rangle$ by subtracting $\langle S^g_3 \rangle$. This gives another support to Ji's procedure advocated in [11] to define and extract the gluon orbital angular-momentum contribution to the nucleon spin. On the other hand, no such measurement is known yet for extracting the quark and gluon orbital angular-momenta $\langle L^{q\prime\prime}_3 \rangle$ and $\langle L^{g\prime\prime}_3 \rangle$ appearing in the decomposition (72). This also means that we do not have any experimental means to separate the contribution of the potential angular momentum

to the nucleon spin. (We point out that a toy model analysis recently made by Burkardt and BC [27] may be thought of as a theoretical challenge to estimate the magnitude of this potential angular-momentum term.)

IV. SUMMARY AND CONCLUSION

It has been widely recognized by now that the decomposition of the nucleon spin is not necessarily unique. In fact, this led to several proposals for the nucleon spin decomposition. They are the Jaffe-Manohar decomposition, the Ji decomposition, the Bashinsky-Jaffe decomposition, and the new decomposition by Chen et al. Since the Jaffe-Manohar decomposition can now be thought of as a gauge-fixed form of either of the Bashinsky-Jaffe decomposition or the decomposition proposed by Chen et al., one can say that these three belong to the same family from the physical viewpoint, i.e. except for unphysical gauge degrees of freedom. On the other hand, the new decomposition proposed in the present paper and the Ji decomposition fall into another family, although the former accomplishes full gauge-invariant decomposition of the nucleon spin including the gluon part, which is given up in the latter.

As fully explained in the present paper, the critical difference between the two types of nucleon spin decompositions is concerned with the treatment of the quark-gluon interaction inherent in the strongly coupled gauge system. We have taken out this term in a gauge-invariant way, and named it the contribution of potential angular momentum as a generalization of Konopinski's potential momentum. Since this part is solely gauge invariant, the gauge principle alone cannot uniquely dictate which part of the decomposition this term should be included into. In the decomposition of Chen et al., this term is combined with the quark orbital angular momentum. On the other hand, in our new decomposition, it is included as a part of the gluon orbital angular momentum. An advantage of our decomposition is that *dynamical* orbital angular momentum and not the *canonical* orbital angular momentum (or its gauge-invariant generalization) appears legitimately in the quark part as is the case in the Ji decomposition. A practical consequence of this advantage is that the quark and gluon orbital angular momenta appearing in the present decomposition can in principle be extracted from the GPD analyses in combination with the analyses of the polarized deep-inelastic-scattering cross sections.

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